AH-1541-CV-19-S

MA/M.Sc. (Previous)

Term End Examination, 2019-20

MATHEMATICS

Paper-III

TOPOLOGY

Time : Three Hours] **[Maximum Marks : 100**

Note : Answer **any five** Question. All Question carry equal marks.

1. **(a) Let U consist $\emptyset$ and all those subsets G of R having the property that to each $x \in G$, there exists $\in > 0$ such that $(x-\in, x+\in) \subset G$ prove that (R,U) is topological space.**
   (b) Prove that cofinite topology on a finite set is the same as the discrete Topology.
2. **(a) Let X be a Topological space. $A \subseteq X$ is closed if and only if $D(A) \subseteq A$**
   (b) Let (x,j) be a Topological space and A, B be subset of x then prove that:-
   (i) $A^0 \subset A$
   (ii) $A \subset B \Rightarrow A^0 \subset B^0$
   (iii) $(A \cap A)^0 = A^0 \cap B^0$
   (iv) $A^0 \cup B^0 \subset (A \cup B)^0$
3. **(a) Prove that a mapping f from a space x into another space y is continuous if and only if Let $f[\overline{A}] \subset \overline{f(A)}$, for every $A \subset X$**
   (b) Prove that Homeomorphism is an equivalence relation in the collection of all topological spaces.
4. **(a) Prove that A topological space x is disconnected if and only if there exist a non-empty proper subset of x which is both open and closed in x.**
   (b) Prove that a subset E of R is connected if and only if only it is an interval.
5. **(a) Prove that closed subset of a compact space is compact.**
   (b) State and prove Lebesgue covering Lemma.
6. **(a) Prove that every second countable space is a Lindelof space.**
   (b) show that (R,U) is T_3 spece.
7. **State and prove Tietze extension theorem.**
8. **(a) Prove that each projecton map $\pi\lambda$ is an open map.**
   (b) state and prove Tychonoff theorem.
9. **(a) Prove that the topological product of a countable family of metrizable space is metrizable.**
   (b) State and prove urysohn metrization theorem.
10. **(a) Prove that a Topological space (x,j) is hausdorff if and only if every net in X can converge to at most one point.**
    (b) Prove that every filter base on a set X is contained in an ultra filter on X.